BOND GRAPHS IN DYNAMIC SYSTEMS DESIGN:
CONCEPTS FOR A CONTINUOUSLY VARIABLE TRANSMISSION

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ABSTRACT

Dynamic systems design requires configurations of component parts to provide specified relationships between dynamic variables. In the case of the continuously variable transmission (CVT) the input-output speed ratio is desired to be continuously variable between an upper and lower limit. Current design of the CVT requires such requirements as meticulously designed and manufactured belts and conical gear sets. Limits on power transmission and reliability are large concerns. Here, concepts for CVT's are developed to produce variable speed ratios where Bond Graphs are a key tool in generating specified dynamic relationships and in gaining insight into the design possibilities and limitations. Designs generated include a serial gyrator configuration and a mechanical filter design that behaves as a high pass filter. Loopic designs are also examined as they give increased freedom in selecting the performance of the dynamic configuration concepts.

BACKGROUND

The theme of this work is the applicability of Bond Graphs to the conceptual or configurational design of a continuously variable transmission (CVT). The continuously variable transmission is chosen as a prime example of a dynamic engineering system with general specifications that fit in a Bond Graph framework, namely effort, flow and power control. A CVT is required to transfer power with sufficient efficiency while continuously varying the flow variable through a given range.

Bond Graphs have been used in conceptual design for the last ten years. Some work has been in using Bond Graphs for generating alternative designs since any part of a Bond Graph can be physically realized in a number of different energetic domains. Finger and Rinderle (1989) and Redfield and Krishnan (1993) deal at least in part with finding equivalent systems to known designs to "spark" novel solutions. Designing systems based on dynamic performance specifications involves specifying input-output requirements and formulating systems or subsystems of given systems to meet the requirements. This is the mechanical realization of network synthesis, Baher (1984). Ulrich and Seering (1989) use static elements to generate steady state designs and Redfield (1992) and Redfield and Krishnan (1993) synthesize dynamic elements to meet frequency response specifications for novel dynamic systems.

Continuously variable transmissions have been on the drawing board for at least the last hundred years. The advantages are well known; continuously variable gearing allows for precise impedance matching between engine and final drive resulting in maximum power transmission efficiency and emission control. They offer a smoother ride than standard or automatic transmissions because no discrete gears changes occur. Disadvantages are typically in the implementation aspects of the designs. Torque transmitting capability, and reliability and material issues are problems to overcome.

CVT's have developed on three basic principles: variable cone belt drive systems, friction drive, and variable stroke configurations. Gott (1991) gives a thorough history and conceptual development of CVT's and shows how CVT's have slowly worked their way into reliable performers for lower power applications. What is a little interesting is that even after 100 years, the basic concepts for continuously variable transmission have not changed. The work of this paper is to develop novel concepts for a CVT based on configuring dynamic system elements to allow variable speed (flow) ratios while being efficient with power.

The rest of the paper steps through using Bond Graphs to generate novel design concepts for a CVT. We start with the idea of equivalent systems and advance toward system synthesis. Each generated concept is examined in little to moderate detail. Along the way concepts are suggested that may be new or old, and some that may be worth further study. The point is that Bond Graphs aided in generating starting points that would not have been easily possible otherwise.

STANDARD APPROACHES

In Bond Graph notation, the configuration design of a power transmission system would be shown as in Figure 1. Power enters on bond 1, is transformed through some yet unknown subsystem (**D** for design operator), and is output to bond 2 which is attached to a load impedance operator, I_L. Typically the load impedance would include an inertia and a resistance.
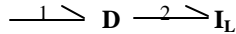
$$\longrightarrow^1 \searrow \mathbf{D} \longrightarrow^2 \searrow \mathbf{I_L}$$

Figure 1 – Representation of power transformation

A first thought for a variable transmission, knowing that transformers modulate effort and flow while conserving power, would be a transformer with an variable modulus. This is the standard approach to the design of these devices and involves clever concepts such as connecting variable radii pulleys with belts and wheels. Figure 2 is the bond graph representation of this approach.

$$\longrightarrow^1 \searrow \mathbf{TF} \longrightarrow^2 \searrow \mathbf{I_L}$$
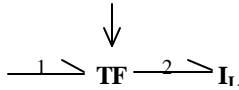
Figure 2 – Transformer with variable modulus

An example of a variable transformer is a traction type CVT by Cleghorn (1996) which is an off-center toroidal drive nearly identical to that in Kraus (1976). (See Figure 3)
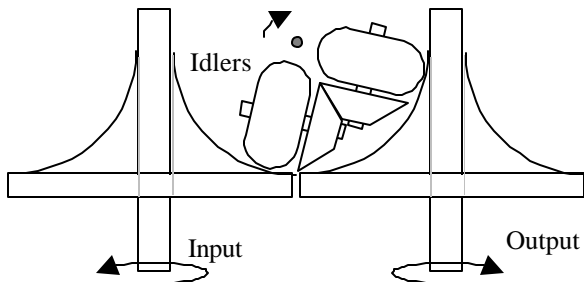

Figure 3 – Off-center toroidal traction drive

UNORTHODOX CVT APPROACHES

Alternative CVT configurations must replace the transformer of Figure 2 with an effectively equivalent system that preferably conserves power while allowing the flow ratio between bonds 1 and 2 to vary continually within some range. It is obviously best if the transmission successfully transmits "all" frequencies of input, but a relaxation of this requirement may be necessary also. Following are developments of system configurations that provide the performance of a continuously variable transmission.

Serial Gyrator Design

A first thought for alternative CVT configurations is driven by an understanding of equivalent systems and is greatly facilitated by insight afforded by Bond Graphs. One equivalent of a transformer is a pair of gyrators in series. Gyrators conserve power but relate effort to flows through a modulus instead of efforts to efforts and flows to flows as in a transformer. If two gyrators can be connected in series and their moduli varied, a variable transmission is configured.

Mechanical gyrators - A first thought for a gyrator may be the mechanical gyroscope. If a torque is applies to a momentum wheel about an axis orthogonal to the momentum vector, a resulting precession develops where the precession angular velocity is proportional to the input torque. The modulus relating the torque to the angular velocity is the angular momentum of the wheel itself, $t=Hw$. Figure 4 shows the modulii and a probable causality for gyrators in series.
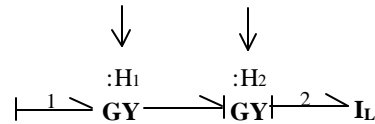

Figure 4 – Mechanical gyrator realization

The transmission ratio would be the ratio of the angular momenta, $\dfrac{f_2}{f_1} = \dfrac{H_1}{H_2}$ so spinning up or down the momentum wheels or changing their mass moments of inertia change the flow ratio.

Certainly a very practical problem is the geometric complication as the transmission rotates. One could envision two gyros oriented about the same center with the output of H_1 connected to the input of the H_2. The more difficult part is tapping into the input of H_1 and the output of H_2, a problem that will not be attempted here.

Electro-mechanical gyrators – A common device that behaves as a gyrator is the motor. Motors and their counterparts, generators, convert current to a proportional torque and vice-versa. The motor constant is a function of both the field and armature and thus can be changed by many variables: field current, electro-magnet geometry, number of effective turns, etc. Thus, a generator-motor pair could create a CVT with a wise choice of modulating parameters (Figure 5). Issues to examine are the power costs in the modulation and the practicality of implementation. For example, the field power for the motor and generator should not be a large portion of the transmitted power or they should be opposite in sign so one could feed the other through appropriate electronics.
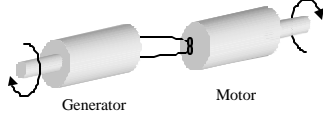
Figure 5 – Generator motor pair with variable field

Hydro-mechanical gyrators – Centrifugal pumps act as gyrational elements converting torque to volumetric flow rate and angular velocity to pressure rise. Series pumps could convert one angular velocity to another with the modulation occurring by varying vane geometry (Figure 6). Issues here are losses intrinsically associated with the pumps.
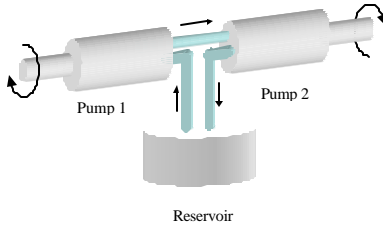

Figure 6 – Series centrifugal pumps

Common Effort Design

A next configuration, in order of Bond Graph complexity, is connecting the input and output through an unknown dynamic system (filter) while requiring the input-output flow ratio to be variable. Since flow ratios are desired, the input and output are connected through a common effort junction as in Figure 7. **D** represents a dynamic subsystem that is designed to allow a variable ratio between flows 1 and 2 while providing adequate power transmission.
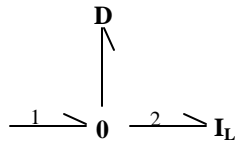

Figure 7 – Design sub-system

Assume the load impedance is linear and due to an inertia, m, and a damping, c, acting on the output flow such that $I_L=ms+c$. With the transfer function generation techniques of Redfield and Krishnan[1993] the transfer function can be generated between the input and output.

$$\frac{f_2}{f_1} = \frac{D(s)}{ms+c+D(s)} \tag{1}$$

We would like to choose a design impedance $D(s)$ such that the flow ratio (for a fixed $D(s)$) is constant for all frequencies.

Single element designs - Equation 1 shows that $D(s)$ cannot be chosen to allow the flow ratio constant for all frequencies but if $D(s)$ is a constant the flow ratio is constant at frequencies below the cutoff of $\frac{c+D}{m}$. In this case D would represent a resistive element and power would be dissipated. If the resistance is such that the lost power is insignificant, the design may be feasible resulting in a low power modulation of higher power transfer. Equation 1 shows that a large resistance is necessary however. A dc (s=0) ratio of zero requires zero resistance but a ratio of ½ would require the design resistance to equal the load resistance. Half the input power would go to each the load and the design. This is not a good situation.

What if the design impedance is that of a compliance, $D(s) = k_d/s$? The flow ratio becomes a second order low pass filter:

$$\frac{f_2}{f_1} = \frac{k_d}{ms^2 + cs + k_d} \tag{2}$$

The dc ratio is one (1) regardless of the choice of compliance so a variable transmission is not realized.

What if $D(s)=m_d\,s$, an inertial element? In this case the flow ratio is

$$\frac{f_2}{f_1} = \frac{ms+c}{(m+m_d)s+c} \tag{3}$$

This is a lag filter with a flow ratio of one at low frequencies, a transition region between the pole and zero, and a constant ratio at high frequencies that could be varied if the design impedance inertia could be varied. If the design inertia was effectively equal to the load impedance, a flow ratio of ½ would develop.

Obstacles to overcome in this design are the frequency problem and the variable inertia. The frequency problem is probably the larger of the two. For a CVT that typically operates for constant speeds, a dc to ac converter would be necessary where the ac motion is varied with the filter of equation 3. A slider-crank could serve this purpose. The more difficult problem is the ac/dc conversion where the output constant speed is a function of the ac amplitude. This problem will not be addressed here.

The variable inertia is not as much a problem as it may seem. A rotational mass where all or part of the mass is moved relative to the rotation axis would serve as a variable mass moment of inertia. Even though the concept is feasible, a detailed study of the mechanics of such a device would be necessary to look at the power needs to change the inertia.

Multi-element designs – The design $D(s)$ in Figure 7 can be more complex than a single element and the

choices are large. What narrows the search are the design specifications of dc modulation of power and efficiency of power transfer. Equation 1 shows that any design impedance with a free integrator causes a flow ratio of 1 at low frequency and designs with resistive elements most likely dissipate an unacceptable power. Designs with a free s in the numerator transfer no flow at dc. A library of impedances that can be realized passively can be searched and various designs examined. It turns out that all impedances without a free s must have resistive components. This would require a study the associated power loss in these elements.

More Complex Serial Design

More complex serial designs can be considered as in Figure 8. This arrangement gives more freedom in the design process because multiple independent impedances (D_A and D_B) can be chosen while trying to meet design specifications.
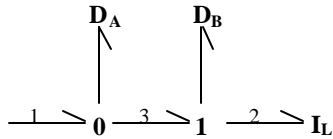
Figure 8 – Multi degree of freedom, serial design.

The flow ratio transfer function is

$$\frac{f_2}{f_1} = \frac{D_A(s)}{(ms+c) + D_A(s) + D_B(s)} \tag{4}$$

For a finite flow ratio at dc, either resistive or capacitive elements are required ($D(s) = c_d$ or k_d/s) since an inertial element impedance ($D(s) = m_d/s$) gives zero f_2 in the steady state. Resistive elements dissipate too much relative power in providing a reasonable flow ratio. The transfer function with capacitive elements becomes

$$\frac{f_2}{f_1} = \frac{k_A(s)}{s(ms+c) + k_A(s) + k_B(s)} \tag{5}$$

Varying the two stiffnesses allows flow ratios of between zero and nearly one with the ratio being independent of the load. This is the good news. The bad news may already be obvious; the flows on the two compliances are constant so they are continually storing energy, not a very practical situation. Also note that exchanging the *0* and *1* junctions in Figure 8 would not give us another design option. D_A would not affect the flow ratio in this case.
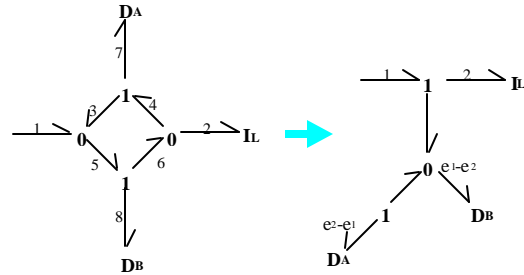
Simple Loopic Design

Bond Graphs with loops between the input and output can also generate designs with more degrees of freedom in impedance choice. The power flow directions
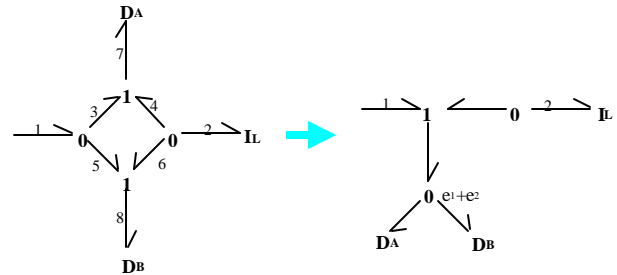
are critical though to whether a new design possibility is being generated. Figure 9 shows three simple loopic structures that, because of power flow orientation, collapse to serial configurations. Thus the consideration we have given to structures such as in Figure 7 still applies.
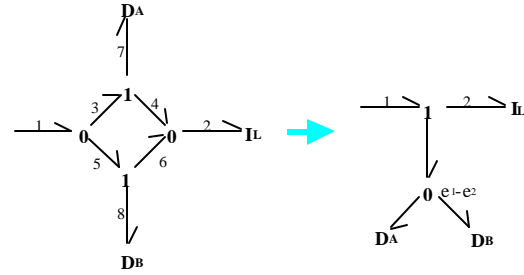
Figure 9 – Loopic configurations that collapse

There is only one simple loopic configuration that is not equivalent to a serial design (Figure 10). Because of its power flow orientation, equivalent structures are still loopic.
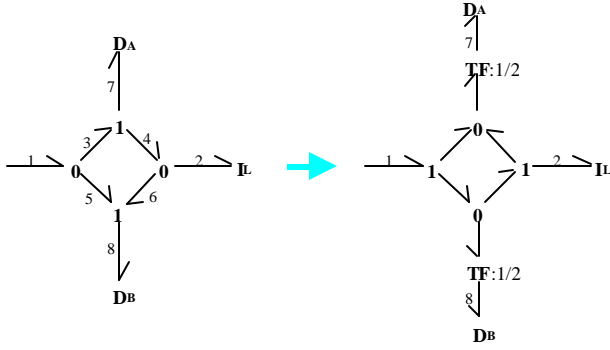
Figure 10 – True loopic configuration

To determine the input – output relationships, chosen variables are related directly from the graph assuming linear impedance operators (D and I). The variables are chosen assuming f_1 is an input, f_2 is the output and the flow on bond 7 and the input and output efforts are worth examining. Equation 6 shows these relationships in matrix form.

$$\begin{bmatrix} -1/D_A & 1/D_A & 1 & 0 \\ 1 & 1 & D_B & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & -I_L \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ f_7 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 \\ D_B \\ 1 \\ 0 \end{bmatrix} f_1 \quad (6)$$

Solving for the output flow gives the transfer function of equation 7. Immediately we can see that the design impedances can markedly affect the steady state performance. Limiting our D's to single port elements we can see that if both are resistive elements, the filter is a first order, low pass and the flow ratio is variable at dc. Significant power is dissipated however.

$$\frac{f_2}{f_1} = \frac{D_B - D_A}{4I_L + D_B + D_A} \quad (7)$$

If both D's are inertial ($D(s)=m_d s$), then the dc flow ratio is zero. Other possibilities can be considered but an interesting one is if both elements are capacitive ($D_A = k_A / s$ and $D_B = k_B / s$). For this case the flow ratio is shown in equation 8 where the load I_L is as before.

$$\frac{f_2}{f_1} = \frac{k_B / k_A - 1}{4(m/k_A)s^2 + 4(c/k_A)s + k_B / k_A + 1} \quad (8)$$

The dc ratio, N, can be varied from −1 to 1 by varying the stiffness ratio ($r=k_B/k_A$):

$$N = \frac{r-1}{r+1} \quad (9)$$

Figure 11 shows this relationship where the choice of r can give forward, neutral, and reverse! And like the configuration of Figure 8, the flow ratio is independent of load.
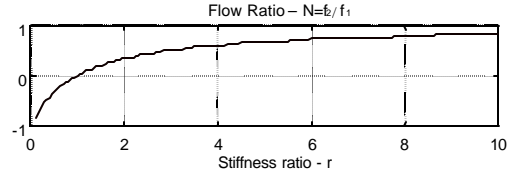

Figure 11 – Flow ratio as function of stiffness

This seems like an exciting result until we step back and realize that, as in an earlier design, the capacitive elements wind up indefinitely thus storing increasing amount of energy. Equation 10 shows this in the transfer function between bond 7 flow and the input flow. In the steady state this flow ratio is equal to the ratio $\frac{k_B}{k_A + k_B}$.

$$\frac{f_7}{f_1} = \frac{2s(ms+c)+k_B}{4s(ms+c)+k_B + k_A} \quad (10)$$

It turns out that the flow ratio for the compliance on bond 8 in the steady state is $\frac{k_A}{k_A + k_B}$. Thus the CVT performs well in terms of controlling the flow ratio, but the efficiency is poor as more and more energy go into potential energy storage. Further study may determine how to tap that energy for useful work or channel it back into the system

Replacing compliances with sources – The above examination can prompt one to consider other possibilities while trying to understand why the compliant elements worked to control the flow ratio. If sources replace the compliant elements to force the flow ratio to a desired value we get the bond graph of Figure 12.
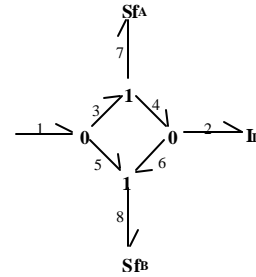

Figure 12 - Flow sources controlling flow ratio

Let N still be the I/O flow ratio. If we demand power in and out equal then $e_1 / e_2 = N$ and a number of relationships develop. The control to I/O flow ratio is still:

$$\frac{f_7}{f_1} = \frac{N+1}{2} \quad \text{and} \quad \frac{f_8}{f_1} = \frac{1-N}{2} \quad (11)$$

The effort ratios become

$$\frac{e_7}{e_1} = \frac{N-1}{N} \quad \text{and} \quad \frac{e_8}{e_1} = \frac{N+1}{N}, \tag{12}$$

and the power ratios become

$$\frac{P_7}{P_1} = \frac{(N+1)(N-1)}{2N} \quad \text{and} \quad \frac{P_8}{P_1} = \frac{-(N+1)(N-1)}{2N} \tag{13}$$

The controlled powers on bond 7 and 8 are opposite in sign as required for power conservation. One question becomes how does the controlling power compare to the input/output power. Equation 13 is plotted in Figure 13 for examination.
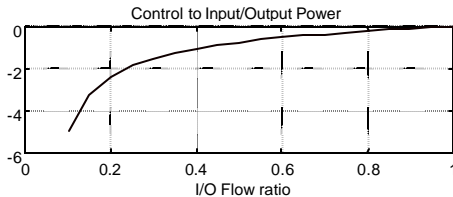

Figure 13 – Control to input power ratio

The main result from Figure 13 is that the controlling flows can control flow ratios from about 0.4:1 with power equal to or less than the input power. For example, a control power of about 21% of the input power can afford a flow ratio of 0.8:1. Since the two sides of the control flows have equal and opposite power, a second stage of the Figure 13 CVT could connect bonds 7 and 8. Two staged gear sets realizing the bond graph in Figure 12 could control the flow with about 4% (0.21*0.21) of the input power. Another stage would reduce this to less than 1% which would allow other concepts in CVT's to control the final stage.

CONCLUSIONS

The work of this paper is to demonstrate the value of using Bond Graphs as a conceptual or configurational design tool for dynamic systems, and in this case, specifically for a continuously variable transmission. A number of concepts were generated, but none were examined in too much detail. Except for a couple of the first concepts, the variable mechanical transformer and the hydro-mechanical design, none of the other concepts are known to the author to have been considered for CVT design.

This does not necessarily mean that any of the concepts will eventually be found feasible but they are concepts none-the-less that are worth further study. Much of design is by redesign and analogy. The Bond Graph approach of this paper gives a tool for novel design for the class of systems where the dynamic relations of the effort, flow, and/or power are the driving requirements.

Some of these CVT systems require further study and perhaps prototyping. An area worth pursuing on the conceptual design side is the inclusion of nonlinear elements.

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